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The Pseudo Specific Heat in SU(2) Gauge Theory : Finite Size Dependence and Finite Temperature Effects

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Abstract

We investigate the pseudo specific heat of SU(2) gauge theory near the crossover point on 4^4 to 16^4 lattices. Several different methods are used to determine the specific heat. The curious finite size dependence of the peak maximum is explained from the interplay of the crossover phenomenon with the deconfinement transition occurring due to the finite extension of the lattice. We find, that for lattices of size 8^4 and larger the crossover peak is independent of lattice size at $\beta_{co} = 2.23(2)$ and has a peak height of $C_{V,co} = 1.685(10)$. We conclude therefore that the crossover peak is not the result of an ordinary phase transition. Further, the contributions to C_V from different plaquette correlations are calculated. We find, that at the peak and far outside the peak the ratio of contributions from orthogonal and parallel plaquette correlations is different. To estimate the finite temperature influence on symmetric lattices even far off the deconfinement transition point we calculate the modulus of the lattice average of the Polyakov loop on these lattices and compare it to predictions from a random walk model.

1 Introduction

The pseudo specific heat C_V of $SU(2)$ gauge theory was already investigated in the beginning of Monte Carlo lattice studies. It is known to have a peak near $\beta = 4/g^2 \approx 2.2$, in the crossover region between strong and weak coupling behaviour. A first finite size analysis by Brower et al. [1] on $4^4 - 10^4$ lattices revealed a strange dependence of the peak on the volume $V = (N_\sigma a)^4$ of the lattice. Here, a is the lattice spacing and N_σ the number of points in each direction. The location of the peak shifts with increasing volume from smaller $\beta-$values to larger and then to smaller ones again; the peak maximum decreases with increasing volume. Such a behaviour is unknown for any ordinary phase transition. The nature and origin of the peak remained therefore unclear, though a connection to the nearby endpoint of the first order critical line in the $(\beta, \beta-\text{adjoint})$-plane was proposed by Bhanot and Creutz [2]: "The peak in the specific heat of the $SU(2)$ model is a shadow of this nearby singularity". Recently, new interest in this extended $SU(2)$ model came up again [3, 4] and a line of second order deconfinement transition points was found, connecting the normal deconfinement transition point at $\beta_A = 0$ to the endpoint of the first order critical line inside the (β, β_A)-plane. The origin of the change from second to first order behaviour and the possibility of a bulk transition have still to be examined.

For both the relation to the extended $SU(2)$ model and the unexplained finite size behaviour in the crossover region a new study of the pseudo specific heat is useful. Moreover, we may now determine C_V with much higher statistics and also on larger lattices than in the early calculations, and we can apply modern analysis techniques. In addition, since symmetric lattices are often used to simulate zero temperature physics, it is important to estimate remaining finite temperature effects which may show up in the pseudo specific heat .

2 Methods to Calculate C_V

We use the standard Wilson action for $SU(2)$

$$S = \beta \cdot \sum_{x,\mu\nu} P_{\mu\nu}(x) \ , \tag{2.1}$$

where

$$P_{\mu\nu}(x) = 1 - \frac{1}{2}\mathrm{Tr}U_{\mu\nu}(x) \ , \tag{2.2}$$

is the plaquette or energy and $U_{\mu\nu}(x)$ is the plaquette link operator. The sum extends over all independent forward plaquettes. There are $N_P = 6N_\sigma^4$ such plaquettes. We denote the lattice average of the plaquettes by P

$$P = \frac{1}{N_P} \sum_{x,\mu\nu} P_{\mu\nu}(x) \ . \tag{2.3}$$

The speudo specific heat is then defined by

$$C_V = \frac{d\langle P\rangle}{d(1/\beta)} = -\beta^2 \frac{d\langle P\rangle}{d\beta} \ . \tag{2.4}$$

There are three methods to determine C_V :

i) one measures the plaquette expectation values $\langle P\rangle$ as a function of β and calculates the numerical derivative at $\beta_M = \beta + \Delta\beta/2$ from

$$C_V(\beta_M) = -\frac{\beta_M^2}{\Delta\beta}(\langle P\rangle(\beta + \Delta\beta) - \langle P\rangle(\beta)) \ ; \tag{2.5}$$

ii) one measures the variance of the plaquettes, which is proportional to C_V

$$C_V = \beta^2 N_P(\langle P^2\rangle - \langle P\rangle^2) \ ; \tag{2.6}$$

or,

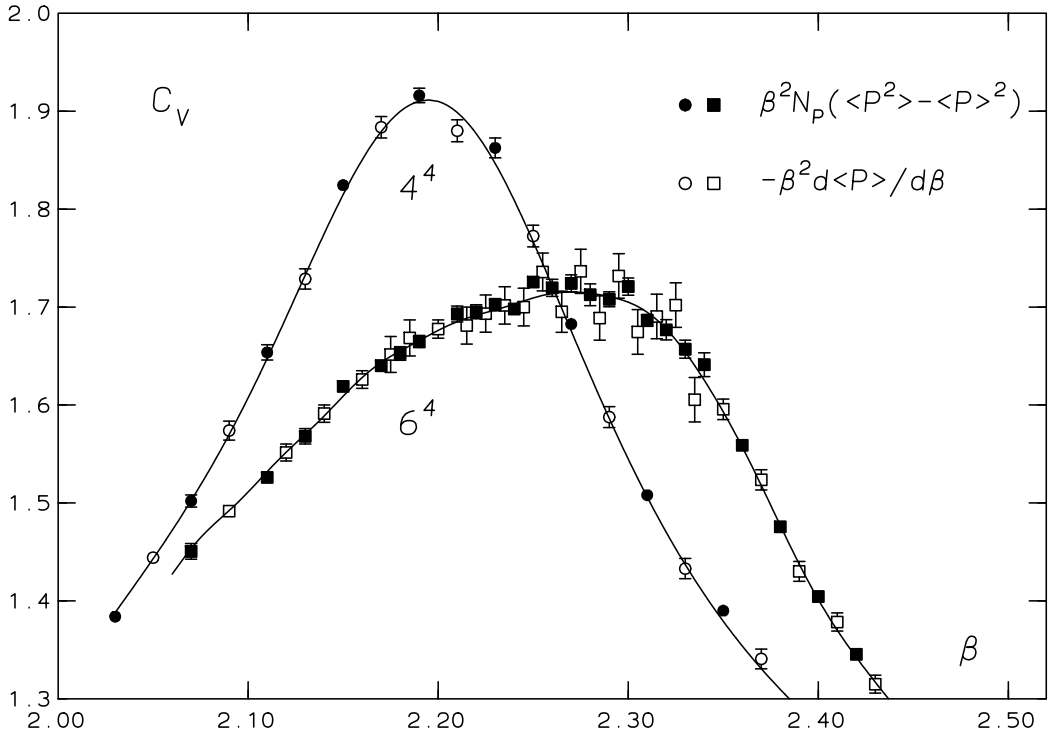

Figure 1: The pseudo specific heat C_V calculated from the variance (filled symbols), the numerical derivative (open symbols) and the DSM interpolation (lines), calculated on a 4^4 (circles) and a 6^4 lattice (squares).

iii) one calculates the sum of plaquette-plaquette correlations

$$C_V = \beta^2 \sum_{x',\mu'\nu'} (\langle P_{\mu\nu}(x) P_{\mu'\nu'}(x')\rangle - \langle P\rangle^2) \ . \tag{2.7}$$

The most straightforward way is, of course, to calculate the variance of $\langle P\rangle$. The density of states method (DSM) [5] may then be used to interpolate between the points. With increasing lattice size the DSM requires however more and more simulation points to obtain a densely populated action histogram. This comes about, because the variance of P is essentially proportional to N_P^{-1} (implying a nearly N_P-independent specific heat C_V, except for the smallest lattices). From

$$S = \beta N_P P \ , \tag{2.8}$$

it follows then, that the width of an action histogram for a fixed $\beta-$value is $\sim N_P^{1/2}$. Compared to that the size of the complete multiple points histogram for the action is

of order N_P. Thus more simulation points are needed for larger lattices to interpolate in the same $\beta-$range. Consequently we have only applied the DSM to the 4^4 and 6^4 lattices.

In Fig. 1 we show the results from methods i) and ii) and the DSM interpolation for these lattices. At each $\beta-$value we took on the average 90-120 thousand measurements. Between the measurements five updates, consisting of one heatbath and two overrelaxation steps were performed, so that the autocorrelation time was of the order of one. As can be seen from Fig. 1 there is complete consistence between the different methods. It is perhaps appropriate at this point to note that if, as it is often done, the plaquettes are measured during and not after each update the resulting plaquette variance is about 20% smaller than expected, because of local correlations among the plaquettes, though the plaquette average is correct.

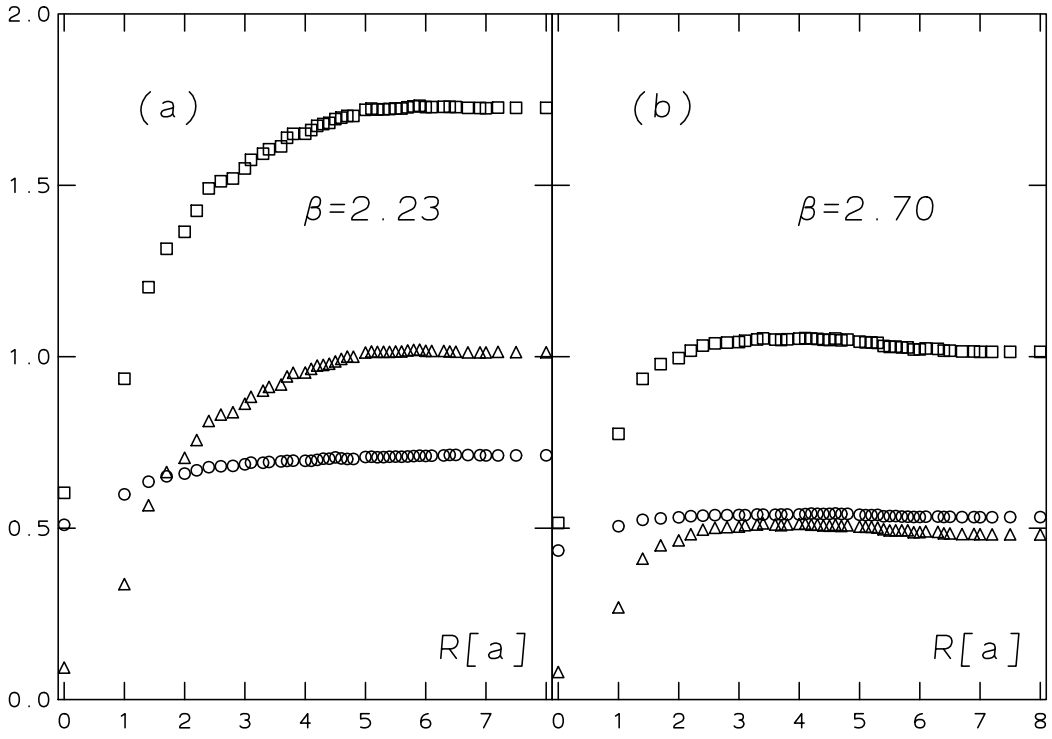

Figure 2: Contributions to C_V on an 8^4 lattice from parallel (circles) and orthogonal (triangles) plaquette correlations up to distance R as a function of R. The sum is plotted as squares. The results are shown for $\beta = 2.23$ (a) and $\beta = 2.70$ (b).

We have also investigated the plaquette correlations. We find in general a rapid fall with $R = x' - x$, the correlation length is of order 1-2 lattice spacings. The pla-

quettes $P_{\mu\nu}(x)$ and $P_{\mu'\nu'}(x')$ in Eq. (2.7) may be in parallel or orthogonal planes. At the peak ($\beta \approx 2.23$) we find that the total contribution of the orthogonal correlations is about 30% higher than that of the parallel correlations, whereas far away from the peak, at $\beta = 2.70$, the contributions are essentially equal. This is demonstrated in Fig. 2, where we compare the contributions to C_V in Eq. (2.7). Shown are the corresponding sums up to distance R as a function of R. Note, that the maximal nontrivial (with respect to the periodic boundary conditions) diagonal distance R on an N_σ^4 lattice is $N_\sigma a$. We see that at the crossover the orthogonal correlations reach their total contribution only at $R \approx 4a$, whereas at $\beta = 2.70$ a distance of $R \approx 2a$ is sufficient. In contrast to that the parallel correlations have a much shorter range and distances of $R > 1.5a$ play no role, both at the crossover and at higher $\beta-$values. There is also no difference among those parallel correlations where the two plaquettes are in the same plane or in different parallel planes.

3 Finite Size Dependence of C_V and Finite Temperature Effects

In Fig. 3(a) we compare the results for C_V from lattices with $N_\sigma = 4, 6, 8, 12$ and 16. The general behaviour already found in [1] is fully confirmed. On the other hand, we see that there is no further finite size dependence in the peak region, if $N_\sigma \geq 8$. This is shown in Fig. 3(b) in more detail for the crossover region.The results suggest, that the finite size dependence of the smaller lattices is related to a different phenomenon. Indeed, the critical point for the $N_\tau = 4$ finite temperature deconfinement transition is at $\beta_c = 2.30$, very close to the crossover peak positon. Since we are using periodic boundary conditions for all directions, the approach to the critical point corresponding to N_σ will influence the plaquette expectation values. To check this, we have calculated the Polyakov loop

$$L(\vec{x}) = \frac{1}{2}\mathrm{Tr}\prod_{t=1}^{N_\sigma} U_{\vec{x},t} \quad , \tag{3.1}$$

and its lattice average L

$$L = \frac{1}{N_\sigma^3}\sum_{\vec{x}} L(\vec{x}) \quad . \tag{3.2}$$

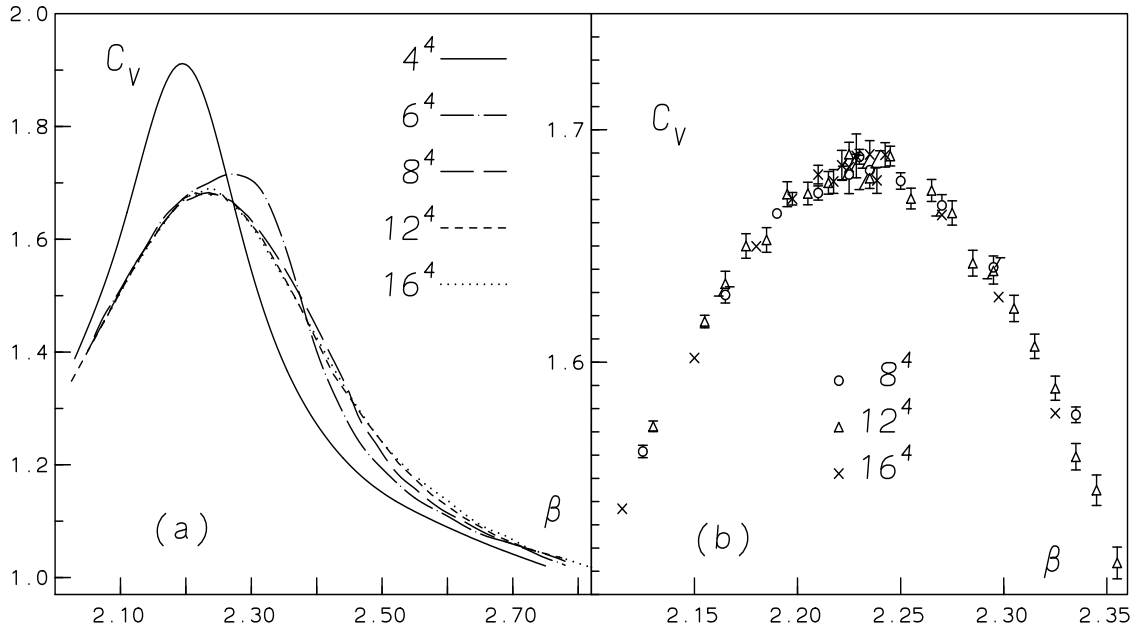

Figure 3: The pseudo specific heat C_V vs. β on N_σ^4 lattices. Part (a) shows the results for $N_\sigma = 4$ and 6 from the DSM interpolation, for $N_\sigma = 8, 12$ and 16 the measured points were connected by lines to guide the eye. Part (b) shows the crossover region for $N_\sigma = 8, 12$ and 16.

As can be seen from Fig. 4, the expectation value of the modulus of L is not zero on symmetric lattices, not even in the strong coupling limit, i.e. we have finite temperature effects also on symmetric lattices. Well below $\beta_c(N_\sigma)$ the quantity $\langle|L|\rangle$ is a constant. With increasing β it starts to increase already before the transition point. It is obvious, that due to the nearby transition points the crossover peaks of the 4^4 and 6^4 lattices are more distorted than those of the larger lattices, where only the right shoulders of the peaks are slightly influenced.

We can actually calculate the $N_\sigma-$dependence of $\langle|L|\rangle$ at $\beta = 0$ from a simple random walk model. For $\beta = 0$ the Polyakov loops $L(\vec{x})$ at the N_σ^3 positions $\vec{x}$ are a set of equal random variables. The expectation value of the modulus of their sum is then assumed to behave as

$$\langle|\sum_{\vec{x}} L(\vec{x})|\rangle \sim (N_\sigma^3)^{1/2} \quad , \tag{3.3}$$

for large N_σ, so that

$$\langle |L| \rangle_{\beta=0} = c \cdot N_\sigma^{-3/2} \quad , \tag{3.4}$$

where c is a constant. Indeed, a simulation at $\beta = 0$ confirms this relation in detail.

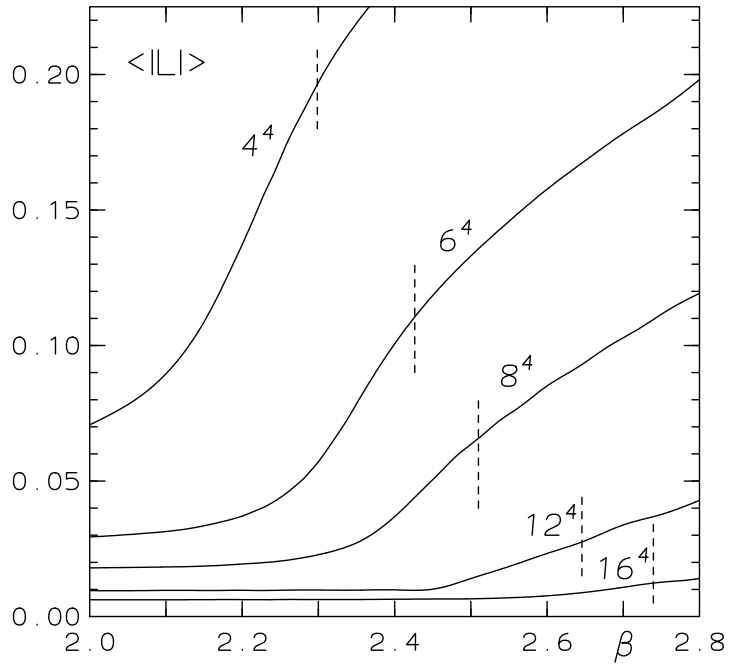

Figure 4: The expectation value of the modulus of L for $N_\sigma = 4, 6, 8, 12$ and 16 vs. β. The broken vertical lines show the locations of the corresponding finite temperature phase transitions.

The logarithms of the $\langle |L| \rangle-$values, which we calculated at $\beta = 0$ are shown in Fig. 5 together with the corresponding results for $\beta = 2$. At this $\beta-$value all lattices apart from the 4^4 lattice have already reached their respective strong coupling limits. Deviations from the limit indicate then the onset of finite temperature effects. One may thus test, in which $\beta-$region results from a specific symmetric lattice can be used to simulate zero temperature physics. For $SU(2)$ a fit to the $\beta = 0$ data leads to $c = 0.400(1)$. The relation Eq. (3.4) holds as well in $SU(3)$ gauge theory and we find there the value $c = 0.296(2)$. A first estimate of the proportionality constant c is obtained from the expectation value of the modulus of a single $L(\vec{x})$, i. e. the

result for $N_\sigma = 1$. For $SU(2)$ it is

$$\langle |L(\vec{x})| \rangle_{\beta=0} = 4/3\pi \quad , \tag{3.5}$$

which differs from c only by 5%.

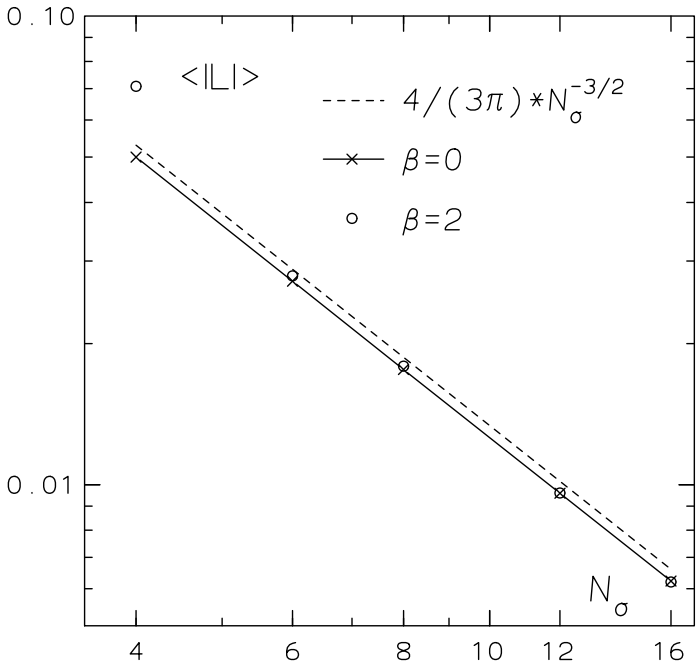

Figure 5: The logarithm of $\langle |L| \rangle$ vs. $\ln N_\sigma$ on $4^4 - 16^4$ lattices at $\beta = 0$ (solid line), at $\beta = 2$ (circles) and from an estimate from the $\langle |L| \rangle$ value at $N_\sigma = 1$ (dashed line).

Our final conclusion is, that the crossover peak is not the result of an ordinary phase transition. For large lattices the peak is at $\beta_{co} = 2.23(2)$, its height is $C_{V,co} = 1.685(10)$. For small lattices ($N_\sigma \leq 6$) however, the interplay of the crossover phenomenon and finite temperature effects shift and distort the peak considerably.

Acknowledgements

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